BTRY 4830/6830 - Matrix Basics

Following convention, we'll use bold script for vectors (lower case) and matrices (upper case) and vector/matrix symbol above the letters for board use:

$$\mathbf{v} = \vec{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \qquad \mathbf{M}_1 = \vec{M}_1 = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \qquad \mathbf{M}_2 = \vec{M}_2 = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}$$

We will also follow statistics convention where the first subscript will index rows and the second will index columns (note this is usually reversed in mathematics literature).

Matrix sum: $\mathbf{M}_1 + \mathbf{M}_1 = \begin{bmatrix} m_{11} + m_{11} & m_{12} + m_{12} \\ m_{21} + m_{21} & m_{22} + m_{22} \end{bmatrix}$

Matrix transpose: $\mathbf{M}_2^{\mathrm{T}} = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$

Scalar times a matrix: $c\mathbf{M}_1 = \begin{bmatrix} cm_{11} & cm_{12} \\ cm_{21} & cm_{22} \end{bmatrix}$

Matrix multiplication:

$$\mathbf{M}_1\mathbf{M}_1 = \begin{bmatrix} m_{11}m_{11} + m_{12}m_{21} & m_{11}m_{12} + m_{21}m_{22} \\ m_{21}m_{11} + m_{22}m_{21} & m_{21}m_{12} + m_{22}m_{22} \end{bmatrix} \quad \mathbf{M}_2\mathbf{M}_1 = \begin{bmatrix} am_{11} + dm_{21} & am_{12} + dm_{22} \\ bm_{11} + em_{21} & bm_{12} + em_{22} \\ cm_{11} + f m_{21} & cm_{12} + f m_{22} \end{bmatrix}$$

$$\mathbf{v}\mathbf{v}^{\mathrm{T}} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \begin{bmatrix} v_1 & v_2 \end{bmatrix} = \begin{bmatrix} v_1 v_1 & v_1 v_2 \\ v_2 v_1 & v_2 v_2 \end{bmatrix}, \quad \mathbf{v}^{\mathrm{T}}\mathbf{v} = \begin{bmatrix} v_1 & v_2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = v_1 v_1 + v_2 v_2$$

If the following holds: $\mathbf{v}_1^{\mathrm{T}}\mathbf{v}_2 = \begin{bmatrix} v_1 & v_2 \end{bmatrix} \begin{bmatrix} v_3 \\ v_4 \end{bmatrix} = 0$ then $\mathbf{v}_1$ and $\mathbf{v}_2$ are orthogonal.

The identity matrix is defined as follows: $\mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, i.e. diagonal elements are "1" and all other elements are "0".

The inverse of a matrix $\mathbf{M}^{-1}$ has a structure such that is satisfies the following relationship (for a "square", *k* x *k* matrix): $\mathbf{M}\mathbf{M}^{-1} = \mathbf{I}$ and $\mathbf{M}^{-1}\mathbf{M} = \mathbf{I}$.

A symmetric matrix has the following structure: $\begin{bmatrix} v_{11} & a & b \\ a & v_{22} & c \\ b & c & v_{33} \end{bmatrix}$.